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NEW YORK [illegible]

Entered as sec[illegible]
Yonkers, N. Y., und[illegible]
Special rate of [illegible]
authorized Novem[illegible]

MATHEMATICS TEACHER

THE OFFICIAL JOURNAL OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

April 4, 1924 EM.

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

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Truan Press, Yonkers, N. Y., and The Lincoln School of Teachers College, 646 Park Ave., New York City

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SUBSCRIPTION PRICE $2.00 PER YEAR (eight numbers)

Educ. library

April 4, 1924 EM

THE MATHEMATICS TEACHER

VOLUME XV JANUARY 1922 NUMBER 1

THE HUMAN WORTH OF RIGOROUS THINKING*

By Professor CASSIUS J. KEYSER
Columbia University

The question I purpose to discuss briefly is this: how much mathematical discipline is essential to the appropriate education of men and women as *human* beings?

This very important question admits of a definite answer and it admits of it in terms of a supremely important and incontestable general principle. Before stating the principle I will give it a name. I shall call it the Principle of Humanistic Education as Distinguished from what is called Industrial or Vocational Education. I say "as distinguished from" because the two varieties of education differ widely in the conceptions at the heart of them and in the motives that actuate them.

In order to set the principle of humanistic education in clear light, let me indicate briefly the obvious facts lying at its base and leading naturally to its formulation. What the individuals composing our race have in common falls into two parts: a part consisting of those numerous instincts, impulses, propensities and powers that we humans have in common, not only with one another, but with many of the creatures of the animal world—a *sub*-human world**; and a second part consisting of such instincts, impulses, propensities and powers as are distinctively human. These latter constitute our Common Humanity. They present an endless variety of detail but in the long course of man's experience with man he has learned to group them, in accordance with their principal aspects, into a small number of familiar classes. And so the nature of our common humanity is fairly well characterized by saying that human beings as such

* Short address by Professor C. J. Keyser at the meeting, December 2, of the New York Section of the Association of Teachers of Mathematics of the Middle Atlantic States and Maryland.

** It is the duty of every teacher and parent to read and ponder Korzybski's "Manhood of Humanity" (E. P. Dutton & Co.), where it is shown that humans are not a species of animal.

possess in some recognizable measure such marks as the following: a sense for language, for expression in speech—the literary faculty; a sense for the past, for the value of experience—the historical faculty; a sense for the future, for natural law, for prediction—the scientific faculty; a sense for fellowship, co-operation, and justice—the political faculty; a sense for the beautiful—the artistic faculty; a sense for logic, for rigorous thinking—the mathematical faculty; a sense for wisdom, for world harmony, for cosmic understanding—the philosophical faculty; and a sense for the mystery of divinity—the religious faculty. Such are the evident tokens and the cardinal constituents of that which in human beings is human. It is essential to note that to each of these senses or faculties there corresponds a certain type of distinctively human activity—a kind of activity in which all human beings, whatever their stations or occupations, are obliged to participate. Like the faculties to which they correspond, these types of activity, though they are interrelated, are yet distinct. Each of them has a character of its own. Above each of the types of activity there hovers an *ideal* of excellence—a guardian angel wooing our loyalty with a benignant influence superior to every compulsive force and every authority that may command. Nothing more precious can enter a human life than a vision of those angels, and it is the revealing of them that humanistic education has for its function and its aim. Stated in abstract terms the principle is this: Each of the great types of distinctively human activity owns an appropriate standard of excellence; it is the aim and function of humanistic education to lead the pupil into a clear knowledge of these standards and to give him a vivid and abiding sense of their authority in the conduct of life.

It is plain that this conception stands in sharp contrast with the central idea of vocational or industrial education. For humanistic education has for its aim, as I have said, the attainment of excellence in the great matters that constitute our common humanity. On the other hand, industrial education is directly and primarily concerned with our individualities, and so it might more properly be called individualistic education. It regards the world as an immense camp of industries where endlessly diversified occupations call for special propensities, gifts

and training. And so its aim, its ideal, is to detect in each youth as early as may be the presence of such gifts and propensities as tend to indicate and to qualify him for some specific form of calling or bread-winning craft; then to counsel and guide him in the direction thereof; and finally, by way of education, to teach him those things which, in the honorable sense of the phrase, constitute "the tricks of the trade."

What are we to say of it? The answer is obvious. Industrial or vocational education, rightly conceived, is essentially compatible with the humanistic type; it may breathe the humanistic spirit; the two varieties of education are essential to constitute an ideal whole, for human beings possess both individuality and the common humanity of man. Industrial education, when thus regarded as supplementary to humanistic education, is highly commendable; but when it is viewed as an equivalent for the latter or as a good-enough substitute for it, it is ridiculous, vicious and contemptible. For the fact must not be concealed that a species of education which, in producing the craftsman, neglects the man, is, in point of kind and principle, precisely on a level with that sort of training which teaches the monkey and the bear to ride a bicycle or the seal to balance a staff upon its nose or to twirl a disc.

It is plain that *one* of the great types of distinctively human activity—perhaps the greatest of all the types—is what is known as Thinking. It consists in the handling of ideas as ideas—the forming of concepts, the combining of concepts into higher and higher ones, discerning the relations among concepts, embodying these relations in the forms of judgments or propositions, ordering these propositions in the construction of doctrine regarding life and the world. It is essential to the argument I am submitting to keep steadily in mind that this kind of activity, our sense for it, our faculty for it, the needs to which it ministers, the joy it gives, and the obligation it imposes are part and parcel of what we have been calling our common humanity. Thinking is not indeed essential to *life*, but it is essential to *human* life. All men and women as human beings are inhabitants of the *Gedankenwelt*—they are native citizens, so to speak, of the world of ideas, the world of thought. They must think in order to be human.

And now what shall we say is the ideal of excellence that hovers above thought-activity? What is the angel that woos our loyalty to what is best in that? What is the muse of life in the great art of thinking? An austere goddess, high, pure, serene, cold towards human frailty, demanding perfect precision of ideas, perfect clarity of expression, and perfect allegiance to the eternal laws of thought. In mathematics the name of the muse is familiar: it is Rigor—Logical Rigor—which signifies a kind of silent music, the still harmony of ideas, the intellect's dream of logical perfection.

Can the dream be realized? I am well aware that most of the things which constitute the subject matter of our human thinking—that most of the things to which our thought is drawn by interest or driven by the exigencies of life—are naturally so vague, so indeterminate, that they cannot be handled in strict accord with the rigorous demands of logic. I am aware that these demands cannot be *fully* satisfied even in mathematics, the logical science *par excellence*. Nevertheless I hold that, as the ideal of excellence in thinking, logical rigor is supremely important, not only in mathematical thinking, but in all thinking and *especially* in just those subjects where precision is least attainable. Why? Because without that ideal, thinking is without a just standard for self-criticism; it is without light upon its course; it is a wanderer like a vessel at sea without compass or star. Were it necessary, how easy it would unfortunately be to cite endless examples of such thinking from the multitudinous writings of our time. Indeed, if the pretentious books produced in these troubled years by men without logical insigh, or a sense of logical obligation were gathered into a heap and burned, they would thus produce, in the form of a bright bonfire, the only light they are qualified to give.

Now it so happens that the term mathematics is the name of that discipline which, because it attains more nearly than any other to the level of logical rigor, is better qualified than any other to reveal the prototype of what is best in the quality of thinking as thinking. And so, in accord with the principle of humanistic education, we have to say that the amount of mathematical discipline essential to the appropriate education of men and women as human beings, is the amount necessary to give

them a fair understanding of Rigor as the standard of logical rectitude and therewith, if it may be, the spirit of loyalty to the ideal of excellence in the quality of thought as thought.

Such is, in brief, my answer to the question with which I began. It is, you observe, a qualitative answer in terms of a great human ideal and a sovereign principle of education. If I must add a word touching the strictly quantitative aspect of the question, if I must, that is, attempt to estimate the extent of courses and the length of time necessary and sufficient to yield the required quality and degree of training, I do so with less confidence and far less interest. For so much, so very much, depends on the pupil's talent and the quality of the instruction. A considerable degree of native mathematical talent is much more common than is commonly supposed. Born mathematical imbeciles are not numerous. I venture to say, regarding time and extent of courses, that, for pupils of fair talent, a collegiate freshman year or even a high school senior year of geometry and algebra, if the subjects be administered in the true mathematical spirit, with due regard to precision of ideas and to the exquisite beauty of perfect demonstration, is sufficient to give a fair vision of the ideal and standard of sound thinking.*

* Readers will be interested to know that the foregoing remarks are but a fragment of the opening chapter of Professor Keyser's forthcoming book, "Mathematical Philosophy" (E. P. Dutton & Co.)—Ed.

THE NATURE OF ALGEBRAIC ABILITIES*

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THE PRESENT CONCEPTION OF ALGEBRA AS A SCHOOL SUBJECT

During the generation from 1880 to 1910 which witnessed the popularization of high schools in America, algebra[1] became fixed as a required first year study, and with a content which I shall call for convenience the "older" content, or the "older" algebra. The "older" algebra sought to create and improve the following abilities: to read, write, add, subtract, multiply, divide, and to handle ratios, proportions, powers and roots with negative numbers and literal expressions, to "solve" equations and sets of equations, linear and quadratic, and to use these techniques in finding the answer to problems. These abilities were interpreted very broadly in certain respects and very narrowly in others. If anybody had asked Wentworth, for example, what negative numbers and literal expressions the pupil should be able to add, he would probably have answered, "Any"; and the pupils did indeed add an enormous variety, including many which were never experienced anywhere in the world outside of the school course in algebra.[2] On the other hand, decimals were very rarely used, and angles were almost never added, in spite of the definite need for that ability in the geometry of the following year.

The actual content with which these abilities were trained was determined largely by two forces. The first was faith in indis-

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* The studies reported in this article were made possible by a grant from the Commonwealth Fund.

[1] Throughout these articles "Algebra" will be used for what is commonly called in this country "Elementary Algebra."

[2] For example:

1. Add: $4x^4y^5z^6 - 3x^3y^4z^5 + 17x^2y^3z^4 - 8xy^2z^3$; $14x^2y^3z^4 + 4xy^2z^3 + 5x^3y^4z^5 - 3x^4y^5z^6$; $-4x^4y^5z^6 - 2x^3y^4z^5 + 4xy^2z^3 + 19x^2y^3z^4$; $2x^3y^4z^5 + 5xy^2z^3 - 7x^4y^5z^6 + 9x^2y^3z^4$; $-12xy^2z^3 + 4x^4y^5z^6 - 15x^2y^3z^4 - x^3y^4z^5$; $3x^4y^5z^6 + 41x^2y^3z^4 - x^3y^4z^5 + 7xy^2z^3$.

Pg. 51, ex. 11; Wentworth: Elementary Algebra. Edition 1906; (reprinted from older editions).

criminate thought and practice—the resulting tendency being to have the pupil add, subtract, multiply and divide, anything that could be added, subtracted, multiplied or divided; and to have him solve any problem that the teacher could devise. The second was the inertia of custom, the resulting tendencies being, among others, to make algebra parallel arithmetic, to continue puzzle problems, to use applications conceived before or apart from the growth of quantitative work in the physical sciences, and to be unappreciative of graphic methods of presenting facts and relations.

The faith in indiscriminate reasoning and drill was one aspect of the faith in general mental discipline, the value of mathematical thought for thought's sake and computation for computation's sake being itself so great that what you thought about and what you computed with were relatively unimportant.

The paralleling of arithmetic was perhaps most noticeable in the order of topics, and in the almost monomaniac devotion to problems with one particular set of quantities and conditions so that there was some one number as the "answer." There was no reason why $a \times a = a^2$ and $a \times a^2 = a^3$ should not have been taught before $a + a = 2a$, but to do so probably never even occurred to the generation of teachers in question. That a general relation as an answer was a much more important matter than the number of miles a particular boat went, or the number of dollars a particular boy had, and more suitable as a test of algebraic achievement—this again hardly entered their minds.

This older algebra survives in whole or in part in some courses of study, instruments of instruction, and examination procedures. As the accepted view of leaders in the teaching of mathematics and in general educational theory, it is, however, now a thing of the past. I shall use the word "algebra" from now on to refer to the algebra which these leaders recommend as content for teaching in grade nine (sometimes grades nine and ten), or as a part of the mathematics of grades eight, nine and ten.

These leaders are not, of course, in exact agreement concerning details of content and degrees of emphasis, but, approximately, they would subtract from and add to the "older" algebra as follows:

They would omit such computations as occur never or very seldom outside of the older algebra. Addition, subtraction, multiplication and division with very long polynomials, special products except $(a+b)^2$ $(a-b)^2$ $(a+b)(a-b)$ $(ax+b)(cx+d)$, the corresponding factorizations, fractions with polynomials in the denominator more intricate than $a(b+c)$, elaborate simplifications involving nests of brackets, compound and complex fractions, and rationalizations other than of $\sqrt{a}$, $\sqrt{a}+\sqrt{b}$, $\sqrt{a}-\sqrt{b}$, L. C. M.'s and H. C. F.'s except such as are obtainable by inspection—all these are taboo except in so far as some emphatic need of the other sciences or of mathematics itself requires the technique in question. Clumsy traditions in ratio and proportion (such as the use of "means," "extremes," "antecedent" and "consequent") are eliminated. Bogus and fantastic problems are forbidden wherever a genuine and real problem is available that illustrates or applies the principles as well. The actual uses of algebra in mathematics, science, business, and industry are canvassed and merit is attached to those abilities which are of service there. The mere fact that an operation, *e. g.*,

$$(2a^4+3a^3b^2c-7bc^2d-8d^2)(a-3b^2)$$

can be performed is not a sufficient reason for asking school pupils to perform it. The mere fact that a problem can be framed is not a proof that pupils will profit from solving it.

Thus from one-fourth to one-half of the time spent on the older algebra is saved. This is used to establish and improve the following abilities:

To understand formulae, to "evaluate" a formula by substituting numbers and quantities for some of its symbols, to rearrange a formula to express a different relation,* to compute with line segments, angles, important ratios, and decimal coefficients, to understand simple graphs, to construct such graphs from tables of related values, and to understand the Cartesian coordinates so as to use them in showing simple relations of y to x graphically.

The discussion of Nunn (1914) and Rugg and Clark (1917), the reports of the Central Association of Science and Mathe-

* i. e., "Changing the subject" of the formula, or "solving" for one of the variables without substituting particular values.

matics Teachers (1919) and the new requirements under consideration by various students of school and college examinations would, if combined into an average consensus, tally rather closely with the foregoing statement.

Whereas the older algebra, giving in the main an indiscriminate acquaintance with negative and literal numbers and their uses, expected an undefined improvement of the mind, this algebra is selective and expects to improve the mind by extending and refining its powers of analysis, generalization, symbolism, seeing and using relations, and organizing data to fit some purpose or question. It expects to improve these greatly for algebraic analyses, generalizations, symbolisms, and relations and for the organization of a set of quantitative facts and relations as an equation or set of equations, and hopes for a profitable amount of transfer to analyses, generalizations, symbolisms, relational thinking and organizations outside of algebra. It expects further to give better special preparation to see the more direct needs for algebra in life at large and to use it to meet them effectively.

This program for algebra is fairly clear and comprehensible, as educational programs go. Neveretheless, a hundred teachers and a hundred psychologists and a hundred mathematicians who should try to act on it as stated, would probably do three hundred things, no two of which would be identical.

We need fuller and more exact statements of the nature of algebraic abilities and of the uses of algebra in mathematics, science, business and industry. In particular we need clearer knowledge of what is, and what should be, meant by "ability to understand formulae," "ability with equations," "ability to solve problems," and "ability to understand, make and use graphs." Still more do we need clearer knowledge of what "analysis," "generalization," "symbolism," "thinking with relations," and "organization" mean.

Ability to Understand and Frame Formulae

The ability to understand formulae may mean simply the ability to understand the face value of the symbols involved. Such is the case when a pupil understands that $A = p + prt$ means

the "amount is equal to the principal plus the product of the principal, rate and time," or, being given the formula and also:

Let the case be one of simple interest, and let the interest accrue without fixed reinvestment,
Let A = the amount in dollars,
Let p = the principal in dollars,
Let r = the percent paid per year for the use of the money, and
Let t = the time in years,

he understands that $A = p + prt$ means "Fill in p, r and t and A will be the correct amount."

The ability to understand formulae may, however, mean the ability to understand the face value of the symbols and also to supply such units and make such interpretation of the situation and the result of using the formula as fits the case and insures the right answer. Thus if, in the case above, the pupil was given only $A = p + prt$, and knew when and how to use it he would really understand much more than the formula. Many pupils, for example, who could translate $A = p + prt$ and use it as they had been taught to do habitually would fail with "What would be the amount of 74 pounds at 1% per month after eight years, the interest being paid every 2 years but left uninvested?" They would not know how to use the formula or even perhaps whether to use it.

The extent to which pupils shall be expected to read between the lines of a formula, knowing when it applies and when it does not, and choosing such units that the result will be correct is a matter of dispute in theory and practice.

On the one hand it is argued that such interpretations are a matter of physics or geometry or business practice or the like, not algebra, and also that the mixture of such interpretations with rigorous mathematical thinking, lessens the instructiveness of the latter. Algebraically, for example, it is correct if $A = p + prt$ and $I = \frac{E}{R}$ to conclude that $AI = \frac{pE + prtE}{R}$. That it happens to be nonsense to say that the amount of money times the current equals the principal times the voltage, etc., is not for the learner of algebra to know or care. So the extremists might argue.

On the other hand, it is argued, first, that algebraic technique divorced from its applications to lengths and weights and dollars and years and amperes and volts is a barren game; second, that absolute clearness and rigor in the statement of formulae so that nothing needs to be read between the lines spoils the best feature of a formula, its brevity. Only two principles are needed, the extremists on this side would say. First, "Use formulae only in ways such as common sense and the facts of the case tell you are reasonable." Second, "Use such units that the answer will be right."

From the point of view of the psychology of the learner either extreme seems tolerable, provided it is operated with consistency and frankness, and provided, in the case of the second plan, too much sacrifice of comprehensibility to brevity is not made. The learner may be taught to insist that every symbol in a formula be defined as a quantity, expressed as a number of such and such units, and to separate sharply his operations with a formula from his choice of which formula. He would then simply refuse to try to operate with most formulae as commonly given. $I = \frac{E}{R}$ would have to be defined as if $I =$ the current in amperes; $E =$ the potential in volts and $R =$ the resistance in ohms,—then $I = \frac{E}{R}$ $A = \frac{1}{2}BH$ for a triangle would have to be extended to:—"Let $B =$ the number of inches in the base of the triangle. Let $H =$ the number of inches in the altitude of the triangle. Let $A =$ the number of square inches in the area of the triangle. Then $A = \frac{1}{2}BH$. If B and H are numbers of feet, A will equal the number of square feet in the area of the triangle," etc., etc. If he chooses the right formula the result of correct computation is ipso facto the right answer.

He may, on the contrary, be taught that most formulae, such as $I = \frac{E}{R}$ or $S = at + \frac{1}{2}gt^2$, are simply hints to guide memory and thought in framing the right choice and arrangement of symbols and numbers, and that he is responsible for that arrangement, and for the interpretation of any results of evaluating or solving it.

The former plan secures abilities easier to learn and requires less skill in the teaching; the latter secures abilities which are perhaps more educative and a better preparation for dealing with formulae as they actually occur in books on science and technology. If so, however, it is because time and thought are spent in the algebra course in learning science and technology, or in solving ambiguities of statement by reasoning out what probably is or should be meant.

The greatest danger in the second plan is in the pupil's framing of formulae. Suppose, for example, that he is told to express in a formula the fact that Profit equals Sales less the Number of Articles Produced times the Production Cost per Article, less Selling Costs plus Overhead, and writes $P = S - NC_p - C_s + O$. Is he to be blamed? His algebra and symbolism are correct. It is only his knowledge of business facts and terms that is at fault. If he writes $P = S - NC_p - C_s - O$, is he to be praised? C_p in actual business may well be a number of cents, not dollars, so that his formula may produce a preposterous answer. Or suppose that he is asked to frame a formula for the number of acres in a rectangular plot, the length and width in feet being given, and writes $A = \dfrac{lw}{a}$. This is true enough if a is correctly defined "between the lines" as the number of square feet in one acre, but of what use is it? In *framing* formulae it seems best to teach the pupil to demand such rigor and adequacy in the conditions given to him that his task is simply translation into an arrangement of numbers and symbols and to demand the same rigor and adequacy of him.

In *reading* formulae it seems reasonable to train the pupil to a certain extent to read between the lines, to be judicious and consistent in his selection of units, and in other respects to use formulae as suggestions and clues rather than as adequate, unambiguous rules. It will be convenient and probably sometimes necessary for him to do so in his actual contacts with formulae in books and elsewhere.

In either case the pupil may profitably understand that from the moment that he begins to operate with the formula until he completes the operations by reaching the desired result or "an-

swer'' all the symbols are simply numbers. Nothing needs to be labeled as inches, feet, dollars, years, volts, ohms, foot—pounds, or the like during the operations. What the quantities are must be considered before operating in choosing or framing the formula, and after operations are done in order to put the right label or interpretation on the "answer." But for the purposes of operation $\text{Amperes} = \frac{\text{volts}}{\text{ohms}}$ is just like $\text{Ans.} = \frac{\text{Number } a}{\text{Number } b}$ or $x = \frac{a}{b}$

Other things being equal, genuine formulae useful in mathematics, science, industry and business are to be preferred for training in understanding ,evaluating, transforming and framing formulae. Other things, especially convenience, are not always equal. The genuine formulae that are of significance to pupils may be too simple, or too much burdened with long numbers, and there may not be enough of them to give the practice considered necessary. So teachers and textbooks tend to make up formulae of just the desired complexity, involving just the relations with which practice is needed, and with just as little or much numerical difficulty as the occasion demands.

The use of these artificial formulae is not essentially more vicious than the use of multiplications like 465×9817. It is probable that not one pupil in a hundred will ever have to multiply 9817 by 465. But we do not object to such work in moderation in arithmetic because the elementary abilities practiced are all useful; and this is a good way to give them practice. In the same way practice with a formula like $P = M^2N + \frac{M(O-N)}{N}$ may be defensible although the formula has only a very slight probability of occurence outside of school.

Ability with Equations

Ability with equations includes two groups of abilities which are, at least psychologically, very different. The first is to manipulate the equation so as to obtain a numerical value for the literal element, or so as to obtain a value for one of the

literal elements in terms of the others. The equation is "solved." The second is to understand the equation as the expression of a certain relation whereby we can correctly prophecy what value a certain element will have, according to the values which one or more other elements have.

Thus $\frac{Q}{2} = KR + 4$ is "solved" for Q by finding that $Q = 2KR + 8$. $\frac{Q}{2} = KR + 4$ is "understood with respect to the relation between Q and R" by understanding that if K is a constant, Q is in direct proportion to R, that if Q is expressed as ordinate to fit R abscissa value, we have a set of points on a straight line cutting the y axis at $+ 8$, with a slope depending on what K is, and that every increment added to R produces, other things being equal, an increment of $2K$ in Q. The older algebra neglected the second ability almost entirely, and even yet the first ability is given far more time and attention in most textbooks, courses, and examinations. Yet the second ability seems of equal or greater importance.

There are three cases of "solving." First, the pupil is taught to organize all the data needed to secure the answer to his problem in the form of an equation with x or n or Ans. or ? or an empty space to be filled, to represent his desired result. "Solving" then means the computation needed to get the x or n or Ans. or ? or empty space on one side of = and to get the other side free from the x or n and where desirable, in simplest form. Sometimes two or more equations with x and y or n_1 and n_2 or Part I of Ans. and Part II of Ans. are used. The ability to manage this organization and manipulation of data is useful. The problems of life, when of this sort, almost never lead to quadratic equations. The computations are rarely literal.

Second, the pupil is taught to "solve" a formula or equation already organized as when he derives a formula for finding the radius of a circle from its circumference, from $C = 2\pi r$.

Third, the pupil is taught to solve equations of the type $y = ax + b$ or $y = x^2 + ax + b$ for a and b, being given related pairs of values of x and y, and to discover the two values of x corresponding to a given value of y in equations of the

type, $y = ax^2 + bx + c$. This third sort of "solving" is valuable if a certain mastery of the "understanding of the relation" has been attained. Otherwise it is dangerously near to being an aimless mental gymnastic. The older but still common practice of solving quadratics only for the case where $y = 0$, out of all relation to the general problem, seems indefensible. The only argument in its favor seems to be that x in $ax^2 + bx + c = 0$ is an unknown quantity and that you should therefore find its value regardless of whether knowledge of its value is of any consequence.

The understanding of equations as the expression of relations, goes straight to the heart of all applied mathematics, showing the formula and equation as the story of a rule or law which certain events in nature follow or approximate; it introduces the most important idea of mathematics, that of quantitative dependence or functionality; it is a vital and potent review of the principle that algebra tells what will happen to *any* number under certain conditions; it furnishes a principle of organization for graphics; it furnishes the treble parallelism between certain important relations, certain graphs and certain equations which will arouse respect for algebra.

It may be retorted that the understanding of an equation as a story of a relation or law is too hard and varied an ability for pupils in the ninth or tenth grade to acquire, in comparison with the more mechanical and uniform "solving." We shall see in a later article that it has been made needlessly difficult by unfortunate usage of terms and unwise building up of certain mental habits which get in each other's way and trip each other up; and we shall there show how to reduce these difficulties greatly.

THE NEXT STEP IN METHOD*

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The topic assigned sets a task impossible for me. To foretell with even approximate accuracy the next step in the teaching procedure of mathematics is certainly beyond my powers. I shall not attempt it. But certain lines of probable development seem to be fairly well indicated from certain general trends of current educational thinking. Possibly some of you, in closer touch with the mathematical field, may from a consideration of these general tendencies be able to effect the best next step. My task this evening, however, is humbler. I shall attempt only a brief survey of what seems to me the pertinent present tendencies.

What are these general tendencies? What suggestions, if any, have they to offer to mathematics?

If I may be allowed to judge, the most widespread and imperative present tendency along methodological lines is the insistent demand that we get our students more fully "into the game." Modern education is increasingly seeing the need of treating pupils more as original centers of energy and of self-directed activity. This is not to deny an essential part in the educative process to adult guidance and control. The problem of combining both essential factors in the highest effectual degree is yet to be solved, and certainly the solution does not lie along the line of giving up control to pupils. But modern education is seeing with increasing clearness that no urge or control merely from without will ever do by or for or with students what is needed to be done. For education to be its real self, the student must somehow from within feel the urge, and from this inner urge become—increasingly as wise guidance makes possible—a real center of self-directed thought and endeavor. Otherwise there is, for me at any rate, small grounds for faith in the procedure. That this means more work from the teacher and a higher order of work is true. That it also means a higher return to all concerned is, if possible, even truer.

* The substance of a talk made before the New York Section of the Association of Mathematics Teachers in the Middle States and Maryland on December 2, 1921.

Teacher and pupil alike have in the past suffered grievously from over much prescription and predigested thought. Both must in a truer sense and higher degree become, as was said above, original centers of energy and self-directed activity.

Let us examine this tendency a little more closely and, as is the wont of modern education, in the light of educational psychology. The study of the psychology of learning has made strides in our day and is now definite enough to demand our serious attention. Two lines of thought seem to have especial significance for us here: purposeful thinking and the psychological as opposed to the merely logical arrangement.

From the point of view of learning, what is the especial value of purposeful thinking and what is its lesson for us? To summarize briefly what I have elsewhere discussed at greater length, the presence of a clear and definite purpose and aim (i) supplies an inner urge for the work at hand, (ii) elicits by the psychological laws of "Readiness" and "Set" fuller pertinent thinking, (iii) supplies an end as guide to thought, (iv) adds to success a greater satisfaction, and so (v) fixes more firmly in mind and character the success-bringing thoughts. This terminology probably sounds strange to many of you and possibly even uncouth, but the facts described are known to everyone who ever found himself "absorbed" in a problem, lost perhaps to his surroundings as he grappled with some "original" in geometry or other engrossing aim. This age-old experience is sweet to all who have tasted it and fair to view in our pupils of whatever age or subject. This experience with its valuable educative results I would extend far more widely.

But this is not all. To purpose the work at hand has always been counted a valuable aid if not a necessary prerequisite to any learning. In and beyond purposing, however, there are many degrees of doing, and what the pupil learns from an experience depends greatly on the degree in which he gets into action. Consider the following as successively richer possible actions: (i) a pupil memorizes the bare words of a demonstration; (ii) a pupil memorizes the idea of a demonstration and can reproduce it in different words; (iii) a pupil makes a given demonstration his own, it becomes his thought, he can use it in a new situation; (iv) a pupil of himself demonstrates a propo-

sition that has been proposed by another; (v) a pupil of himself sees in a situation the mathematical relations dominating it and of himself solves the problem he has thus abstracted from the gross situation. Clearly a strong and definite purpose would help the pupil to success in each of these activities and would with equal certainty help fix the attendant learning the more firmly in his mind, but how different the pupil's experiences in the several instances and how markedly different the learning! Do you tell me that I am proposing the impossible, our pupils haven't the requisite ability? Possibly so, but considering these as five successive steps in an ascending scale, is it not true that the average of our pupils has in the past half century or so moved perceptibly up this scale? Did "originals" a century ago occupy so prominent a place in geometry teaching as they do now? To ask the question is to answer it. Any observed improvement, however, is not in the ability of the children concerned, if anything our pupils now are less a selected lot. No, it is our teaching that has improved, and I for one have faith that we can even yet improve. What I am proposing is that we no longer be content with stage iv as given above, that of solving problems proposed in definite terms by others, but that we move on to stage v and build up in our pupils the ability to analyze with reference to some purpose a total gross situation and to abstract therefrom the mathematical relations implicit in it and necessary to control it. Such thinking is the real thinking of life. To remain content with anything less is to be content that our pupils remain forever in tutelage, that they be henceforth underlings instead of freemen in that realm of thought we wish them to inhabit.

Possibly the discussion of the psychological vs. the logical order of presentation will make clearer the idea advanced above; for the two lines of thinking are in a true sense but correlative aspects of the same point of view. By the psychological order is meant the path the mind takes in individual discovery and experience; by the logical order is meant the scheme of arrangement wherein and whereby the mind prepares for future use the results of its experience and discovery. Suppose you undertake to solve what is to you a new and difficult "original," how many steps do you take? Imagine a faithful record made of

your whole experience of seeking. In such a complex manifold of wanderings—fruitless efforts, valid testing, finally successful effort—you have one psychological arrangement. Now contrast this with the crisp orderly arrangement in which you demonstrate the correctness of your conclusion, and you see how different the psychological and the logical can be for you at that stage of your development along that specific line.

To draw from this conception its lesson for us we must note that as teachers we are concerned not merely with the objective goals reached by pupils, but quite as truly with the actual searchings themselves. Out of a prolonged search a region of thought may be mapped. There is even ground for claiming that only out of such a wandering effort can come that organization of experience we call knowledge to mark it off from mere information. Certainly in a sense and to a degree the proposition is true. And still more, it is out of successful search that successful methods of attack must come and the courage to try. That a personal search and survey is necessary thus really to organize knowledge we cannot too much emphasize. But on the other hand an unaided search may prove unduly costly both of time and zeal. Here is the opportunity of the real teacher: What fields promise rich rewards for my group of pupil-searchers? How can I step in to save their effort from waste of time and undue discouragement and yet leave to them a real and fruitful search? The good teacher of mathematics nowadays knows, perhaps as do few others, that to have searched and found, leaves a pupil a different person from what he would be if he merely understands and accepts the results of others' search and formulation. The acceptance of this principle marks one of the definite advances made in our teaching of secondary mathematics within the past hundred years. But we may fail to realize our full possibilities here. In the ideas of the preceding paragraph this advance might mean no more than progress from stage ii or iii up to stage iv, from a stage where the child merely accepted the thought of another up to a stage where his utmost would be to solve problems formulated and propounded by another. Much more is possible.

The position I wish here to advocate is that we now advance to the next stage. Just as it is no longer sufficient that a pupil

accept Euclid's or Wentworth-Smith's demonstrations, just as personal individual work with "originals" is now seen to be necessary to give the discipline we seek, so let us be no longer content to have our pupils slavishly follow the masters' logically worked out order of thought whether in geometry or algebra, trigonometry or elsewhere. In either case the masters' thoughts may well be better than any possibly to be got out by the pupils themselves. In the one case as in the other the advocated plan may require more time, but in every case it is quite possible to gain apparent time and apparent thought at the real expense not only of zest but actually of appreciation and progress. This conception of the psychological vs. the logical order of presentation has many far-reaching implications, more than I can here even hint at, but I beg you to consider the matter. Let your thoughts play about it. On this conception we are to think of the severely logical arrangement, whether of the individual problem demonstration or of a whole topic arrangement, not at all as the initial point of attack, but as the end-outcome to be obtained only after a long and more or less wandering process. Nor need we fear: we do not make pupils logical by requiring them acceptingly to retrace our final logical formulations. Power comes from exercise of function, and usually is of slow growth. The child begins immature in practically all respects and his works manifest this all-pervasive immaturity. To shut our eyes to the fact, to pretend that it is not so, is not only to stultify ourselves, but actually to hurt our pupils. Power of logic, in the fullness that delights the learned adult, is beyond the child. He must attain it slowly. But after each experience, whether it be of his roguish four-footed playfellow or of such abstract matters as time or space, the child does in fact organize more or less adequately the results of that experience. With each such experience organized into his nervous system, he goes forth by that much a different person. The next experience in that field takes place on a correspondingly higher plane, and the youth emerges with a still more adequate organization of his previous varied successive experiences in that field. The teacher may serve as guide and helper—must as a rule so serve if undue loss is to be avoided—but the actual organization if it gets into the boy at all is put there by the boy, built up in him in and through his own efforts.

Let us now put together four thoughts taken respectively from the four last given paragraphs. First, it is in purposeful activity that we have the necessary definite aim, inner urge, full "ready" thinking, and increased satisfaction necessary—all working together—for effectual, well-organized, and abiding learning. Second, our pupils must learn to face whole situations and to abstract from each such living whole situation the mathematical relationships necessary to its control. Anything less is but abstract and unreal. Third, it is the personal search that counts, on no other basis is there real, personal, and promising organization. And fourth, the logical articulation within a whole topic and between topic and topic must itself have for each pupil a psychological history, be itself a growth, a result slowly attained from a series of successive particular organizations following particular and personal searchings. Else again we lessen the hope and promise of the boy's mathematical future.

These thoughts seems to me at least to suggest the "next step" or perhaps steps in the teaching of mathematics. If the analysis given be accepted, we must try to base our work on purposes accepted as such if not originated by the pupils. This means that we have first to begin where our pupils happen at that time to be in the matter of interests and ideas, and call into play purposes corresponding to interests actually existent. This is, of course, only a beginning. On the present as a foundation we must build up interests fruitful for continued future growing. We need not hope, such is the voice of psychology, to build any abiding and worthwhile interest except upon the foundation of a pre-existing native capacity. To begin where the child is means probably that much if not all of our first mathematics be found in other settings, perhaps of physical, perhaps of commercial or other social phenomena. Within these natural setting situations our pupils will find the needed experience in facing concrete whole situations. They will thus learn to detect mathematical relationships where perhaps on the surface none are at first apparent. Such a procedure is to be sharply distinguished from the common use of illustrations and application. Nowadays the instructor, or most often the textbook writer, first decides on the grounds o logical (demonstration) connectedness that a certain principle shall next be treated; he then hunts about for—

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or better still contrives—some instance where the principle occurs among matters probably known to his prospective pupils. This he does with the idea that the principle will thereby be the more readily learned. What I am proposing is the real thing of which this common procedure is the immitation, often no better than a counterfeit.

The plan proposed is that some mathematically fruitful purpose be evoked among the matters for which the pupils—as they now are—have already strong actual or immediately potential interest. Strength of interest and consequently strength of purpose is a first desideration; from it as we saw will come better and more effective thinking and better learning. At first the pupils will feel, and it will be true, that the mathematics used is purely subsidiary. But each instance where mathematics is thus used will be none the less the best possible application and illustration of that mathematics. If this were all and no progress beyond this were to be made, no teacher of mathematics might be necessary. This, however, is only the beginning. Successive experiences of this kind result in the accumulation of a body of mathematical knowledge and procedure. We recognize it as mathematical and give it some sort of name, but continue for a while longer to think of it solely as instrumental. As instrumental, however, we are compelled to organize it, else we can't use it well. At first we do this informally, though always definitely, but in time we become more and more systematic in our arrangement.

What all of this means by way of giving reality to the boy's mathematical thinking I need not emphasize. Learning and application go thus hand in hand. Perhaps the physics teacher will no longer sneer that he has to teach anew whatever mathematics he needs to use. As from time to time the need to control different types of situations arise new conceptions and procedures of mathematics are introduced. The new type of teacher will not fear, as occasion demands, to open up a vista of the graph of a curve, of limits and variables, or of trigonometric functions, to pupils who from our old logical demonstration order point of view are not yet ready for them. Indeed our teacher will not be restrained by any of our conventional subject divisional lines. Algebra, arithmetic, analytics, solid

geometry, trigonometry, even calculus—all is grist that comes to his mill. For he is not thinking in terms of completed logical organization, but of control of situations. And actual situations do not in life follow each other according to logical sub-divisions. Nor will he be unduly disturbed if some familiar topics by chance fail to appear. Many were better forgot. Our teacher will contrariwise ask first what ideas are needed for coping with this situation. Of any new idea needed, for such control he will ask whether the pupils can get sufficient mastery of the idea to enable them to control the situation. If yes, the teacher will then go ahead and call into play anything reasonably available, past experience, reason, intuition, authority, experiment, anything to create a working conviction and an effective grasp. Demonstration and articulation he will, if need be, leave recklessly to the future.

Does this mean haphazard teaching? Far from it, though it may well mean unpredictability. That is, the particular path of progress will not be fore-arranged or even foreseen. In other words we are basing our order of progress frankly on learning conditions and not on the bare requirements of orderly demonstration articulation. Since pupil interest and purposing are essential factors among conditions and since there are in their precise manifestations among the most unpredictable of mundane affairs, it seems probable that an element of unpredictability is a permanent factor in the proposed "next step." But this is to say nothing less than that we are proposing to introduce the human element into a part of mathematics where hitherto it has been too often lacking. The capable advanced student has always felt the warmth of this factor. Our procedure has too often denied it to the beginner.

But does unpredictability mean therefore no place for training in the teaching of mathematics? By no means. In the first place there is much room for predictability as to what will and will not happen. Humans are after all much more alike than they are different. Moreover, a proper combination of the predictable with the unpredictable is just the factor needed to give zest to all human endeavor. Our games are built on it. Mathematics teaching on the basis herein suggested will require more not less preparation, and be far and away more interesting.

Any two successive years will present such variations and such new possibilities as always to challenge the best available teaching. As the teaching of mathematics now is found, there is a persisting superstition—I could not in this presence call it more—that teachers of mathematics, as distinguished from research students in the field, either die of dry rot or write textbooks or become administrators. On the theory of this superstition teaching as teaching seems not to satisfy. The plan proposed indeed looks to the emancipation of the boy, but looks just as strongly beyond that to the emancipation of the teacher.

If we left the mathematics studied to the merely instrumental aspect as indicated above, our work would be most faulty. It is indeed highly probable—at least so I view it—that for the majority they will not progress much beyond that stage. It is fairly clear, at least their reports so indicate, that even many engineers in their thinking and appreciation never get beyond the strictly instrumental level of mathematics. Far be it from me to intimate that all must find excellence along the road chosen by those who now hear me. It cannot be demanded. It certainly is not to be expected. But there will be some who find themselves here. Many choice souls there are who find delight in the realms of "pure" mathematics. Some sooner reach the confines of their talent than do others, but as far as they go together they enjoy the like pleasures. These will soon begin to differentiate themselves from the others, from the merely instrumental crowd—shall I say from the common herd? The alert teacher will note early in some a certain inclination to dwell longer on the mathematical relations from time to time, disclosed, a certain keener wish to organize the successive results into more systematic form. For these there is born in time a different and distinct interest, a wish to pursue mathematics as mathematics and for its own sake. Obeying this psychological demand a new group must be formed to exploit the newly differentiated interest. But even here psychological methods of approach will dominate, though with gradually expanding psychological outlook larger and larger articulation will be sought, and finer and finer logical points will be attempted. I can well believe that somewhere along the line Euclid's great effort will be studied as it properly deserves. Nor will this

mean a series of lectures giving in pre-digested form what the professor has in his study thought. The students themselves—always under wise guidance—will in this special and limited field face again a concrete living whole situation. How few postulates can we get along with? What would result if we reversed this postulate? Again with trusty weapons of attack forged in many a hard-won battle—the figure, alas, is too small to hold the truth—they will analyze this—for them—living situation, and will again divide the spoil in the end, each with necessary justice keeping all he wins with much his fellows also won. And each such student will in and from such endeavors grow in a fashion denied at any rate to me when I in no mean institution studied mathematics in what was then the approved fashion of attack. Whether now I should regret for my present self the treatment I then received need not concern us. But like the youthful Hannibal of old I swore then at the altar to the cause of truth and fellow-man eternal enmity to that form of teaching which disregards the learner and the paths he needs must take to learn. For this cause I now speak these words.

THE NEXT STEP IN CONTENT IN JUNIOR HIGH SCHOOL MATHEMATICS*

By Professor DAVID EUGENE SMITH
Teachers College

The next step relating to subject matter of the Junior High School is really the first step—for we can hardly be said to have taken that as yet.

The problem resolves itself into recognizing that mathematics permeates every science that we have, every department of business activity, and, indeed, most of the other fields of human interest, and that the first step is to ascertain precisely what phases of mathematics are needed by the average, well-informed citizen in order that he may understand any one of the manifold lines of human activity in which the science plays an important role. We have made a fair beginning in the solution of this problem, but that which constitutes a solution today ceases to be a complete solution tomorrow because of the rapid changes in the applications of mathematics to the various lines of human activity.

No one wishes to preserve the dull, uninteresting, unusable parts of mathematics, and the teachers of the science are today the leaders in replacing this material by that which meets current human needs. It seems indisputable, however, that the average, fairly-well educated American citizen should know such uses of elementary algebra as depend upon the formula, the graph, the negative number, and the simple equation. It also seems evident that he should know the significance of and be privileged to enjoy a subject like intuitive geometry, and that the meaning of the trigonometry of the right triangle should be made clear to him through such common applications as he is likely to meet within the ordinary reading of elementary science. It also seems desirable that he should know the meaning of a geometric proof—the only opportunity that he has to come into close contact with deductive logic in his work in the secondary school.

* Synopsis of an address given at the New York Section of the Association of Teachers of Mathematics in the Middle States and Maryland, December 2, 1921.

The material necessary to accomplish this result is not extensive. It should be and is usually arranged for teaching in psychological order, and for myself I should be glad if it were so planned that the pupil might enter any class without having necessarily passed the preceding work. This would mean that a pupil who fails might substitute another subject by which to make up his deficiency, if he so desired, but that in any case he could proceed with his mathematics. This would assure to every individual a fair all-round knowledge of the essentials which everyone should know, without requiring the impossible in the exceedingly rare case of a pupil who cannot, under a teacher of fair ability, master the elementary work that is covered in the junior high school.

To place all human knowledge upon a dead level of importance, as some of our esteemed friends in the educational world seem to advocate, is hardly worthy of discussion. A few great branches, like mathematics, correlate so closely with every other branch as to make their teaching an imperative requirement, and one purpose of the junior high school is to present such branches to the pupils of the seventh, eighth and ninth school years.

I do not suggest the disciplinary value of mathematics, because the idea that "the doctrine of mental discipline has been exploded" has itself been so thoroughly exploded by the results of the recent inquiry made of a body of leading American psychologists by the National Committee that we may consider the discussion a matter of ancient history. At any rate, however, no one puts forth any claim that this is the determining reason for the teaching of mathematics.

There should be mentioned in this connection, the danger of so humanizing our mathematics as to leave in the pupil's mind a rather large amount of humanity, but no mathematics at all. There should also be mentioned the fact, patent to every successful teacher, that one of our greatest purposes is to reveal to the student something of the soul of our science. It is only with this purpose in mind that we shall succeed in our teaching of what an ancient and worthy Oriental writer called "the science venerable"—a phrase which, etymologically as well as actually, means "the knowledge lovable."

THE NEXT STEP FOR THE ADMINISTRATOR IN JUNIOR HIGH SCHOOL MATHEMATICS*

By Examiner JOSEPH K. VAN DENBURG
New York City Public Schools

From the administrators' point of view the next step in administration of high school mathematics is that of clearing the minds of many old and experienced teachers of the idea that there can be no improvement in secondary school courses in mathematics. As I see it this is the one great barrier to the advancement of the new work.

The junior high school teacher is new to the work, open-minded, and adaptible, and the better work in mathematics is being undertaken in those classes. On the other hand, not so long ago I observed a high school recitation in algebra, where the teacher expressed horror at my suggestion that the pupils might be expected to study algebra from some other reason than from a sense of duty or "to pass the Regents." Fortunately there are not many that hold to the point of view that all work in mathematics must be done from a sense of duty rather than from an appreciation of relative values.

To convert these older and no longer plastic teachers to the newer work is a tremendous problem. The steps in that conversion, as I see them, are:

I. The conversion of the licensing body so that the older teachers may know that the qualifications which they presented are no longer acceptable for those who desire to teach in high schools.

II. The conversion of the principal of the high school to the fact that his school is losing ground if it fails to try out the new work.

III. The conversion of the head of department of mathematics who plans the work in a general way and who may know that he is backed up by the principal seeking important improvements in his courses in mathematics.

IV. The appointment of a liason officer who will have authority as a supervisor in junior and in senior high schools at

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the same time. This supervising officer should make sure that junior high school pupils who have been taught the correlated or general introductory mathematics are not penalized when they reach the high school, and he should arrange to have teachers of the crystallized type visit and observe the work of the children that are doing the newer kind of mathematical work.

V. As a last step, I would propose that the teachers who will not make a study of the newer work and who persistently and consistently refuse to open their minds to new ideas be rated as unsatisfactory in instruction, no matter how well they may be able to teach the kind of work that was required ten or twenty years ago.

MINIMUM MATHEMATICAL REQUIREMENTS FOR AGRICULTURAL STUDY

By PROFESSOR H. B. ROE
University Farm, St. Paul, Minn.

Twelve years of responsibility for the instruction in mathematics in the College of Agriculture and in some measure in the School of Agriculture, may be found some justification for this presentation with considerable definiteness of certain of my own views on this subject.

General Educational Value of Mathematics. It has never been successfully disputed that in any line of study, mathematics develops certain habits of thinking as does no other line of work. The mastery of mathematics demands the development of the three great elements of good thinking power namely: application, concentration, and power of analysis. The first two of these must necessarily result in: clarity, good logic, breadth of thought, exactness and accuracy.

The power of analysis implies development of the power of: comparison, contrast and the recognition of relative values, all of which are essential in advanced agricultural study.

Mathematics the Basis of Scientific Classification. The science of mathematics is essentially a classified field and so forms the basis for classification in all sciences. That is, in large measure, all science is dependent on mathematics for its development. Agriculture is a specialized field of science, classified according to the various phases of work and each of these phases demands more or less careful mathematical training and ability. These phases might be roughly listed as follows: farm management, dairy production and feeding; surveying, drainage and irrigation; buildings, machinery and power; and the special agricultural sciences, including: plant breeding, pathology and economic zoology; agricultural chemistry and soils; and agricultural economics and rural sociology.

Fundamental Personal Requirements in Mathematics for Agriculture. The fundamental personal requirements in the study of mathematics for agriculture are, in the main, two: first, interest and earnestness on the part of the student in the development of his chosen line of work, and second, the teacher.

The strength of the first of these requirements is very largely dependent upon the second, namely the teacher, who must be possessed, not only of a thorough knowledge of his subject and ability to teach it, but also an unlimited enthusiasm and belief in its practical value. He must be ready and willing to recognize and encourage the individuality of the student, never insisting on teaching by the book or any set method.

A. Special Means and Methods for Slow Students

For students slow to grasp mathematical ideas, carefully drawn charts and mathematical models are often a great help. With any type of student geometrical representation of fundamental algebraic law is excellent and helps to fix such law by means of the avenue of the eye as well as of the mind. The teacher must keep constantly in mind the great end for which he is striving, namely, *understood* method and result, the means to that end being only secondary.

Abolish Artificial Classification and Boundaries. It seems to me especially essential in the teaching of mathematics to students of agriculture that we get away as far as possible from the artificial boundaries and limitations which have been given to the different fields of mathematics. That is, we should not cling to the names: arithmetic, algebra, geometry, etc., but rather speak of *mathematics.* There is no sharp dividing line anywhere in the field. We all recognize that algebraic law is fundamental in mathematics, yet we teach arithmetic first. No one can show where arithmetic leaves off and where algebra begins. We should not attempt to practice it in our teaching. The transition from arithmetic to elementary algebra should be gradual and unnoticeable to the elementary student. By similiar gradual transition, we should pass from algebra to geometry and we should use geometrical illustrations as far as possible all the way. What we need in the way of developed courses are *general* courses in mathematics. A good many of our best teachers of the present day are successfully working on just such a plan.

The Extent of Mathematical Training Needed is Dependent upon the Phase of Agricultural Study to be Followed. It is impossible to place an exact time limit on the study of mathe-

matics for any phase of agriculture study to be pursued by the student. There are four major phases, namely: The Business of Farming; Teaching of Agriculture and Allied Work in Regularly Curricularized Courses; University Extension Work in Agriculture; and Scientific Investigations Relative to Agriculture. Let us consider these in order.

The Business of Farming

The present day up-to-date farmer must have in his mental equipment sufficient, well-understood mathematics to enable him to carry out mathematical operations which may come into his work of conducting the farm intelligently and profitably. He should be able to measure up and compute the areas of his various fields and to understand the geometrical methods for straightening their boundaries. If he wishes to paint his barn, he should have the knowledge at hand that would enable him to compute closely the amount of paint he will need for the purpose. He should have a knowledge of the principles of percentage as applied to investments, depreciation, income, loans and capitalization. For example, if he is contemplating an enlargement of his business or going into some new field, he must know something of the value of his time and understanding. Assuming that in his general farming operations, he has made a clear profit of $2,400 for a given year, if money be worth 6%, he should be able to understand that his time should capitalize at $40,000. He should be able to draw simple plans, both of his farmstead and his simpler buildings and he should be able to make up comparative tables or charts, through a series of years, of yields and returns therefrom of different types of crops; and so we might go on. The avenue in our educational system which provides for the business of farming is the agricultural high schools and our schools of agriculture. The recognized entrance requirements for these are the completion of the work of our regularly established eighth grade, calling for a complete course in arithmetic, the high schools and agricultural schools following this up with special advanced courses in what is known as farm arithmetic. I have no patience at all with the idea of requiring long and arduous labor by the farm boy or girl in mastering the intricacies of bank discount, annuities, stocks and

bonds, alligation, and compound proportion. For such a child, looking forward to work in the School of Agriculture, these topics may well be largely omitted and the time given instead with profit to the following.

(a) Extensive practice in the fudamental operations with the numerous and easy short cuts, such as: to multiply by 125, annex three ciphers and divide by 8; to divide by 25, point off two places and multiply by 4; to lay off a right angle form a triangle with 3 lines in the ratio of 3, 4 and 5.

(b) More work in common fractions and the various processes of reduction of same.

(c) The common operations of percentage as applied in problems of simple interest, investments, income, and simple accounting.

(d) The formulas, with applied problems, of elementary mensuration, including: the areas of all simple plane figures such as the triangle, rectangle, parallelogram, circle, sector, segment, and brief rules for short cuts where these are readily available. Illustrations are: area of circle equal 3 1-7 times its diameter; Area of a segment equals its height times the length of the chord, times the sum of $\frac{2}{3}$ and $\frac{1}{2}$ the square of the ratio of the height to the length of the chord; that is, $A = al(\frac{2}{3} + \frac{1}{2}[a/l]^2)$ in which A stands for area; a, the altitude or middle ordinate of the segment and l the length of the chord. (From Eng. News-Record, Vol. 79, Nov. 22, 1917.)

(e) Volumes of simple solids as the cube, prism, pyramid, cone, frustrum of cone or pyramid and the sphere.

(f) The general nature of the right-angled triangle, including square and cube root.

(g) Most important of all, *rigorous and varied training in problem analysis.*

There is no other part of the average student's preparation in mathematics of which we hear more complaint from their instructors in other subjects than of their inability to handle common fractions and decimals. It is pitiful, too, when an instructor in carpentry gives the dimensions and shape of a building to find that an otherwise capable student is utterly unable to compute the paint required to paint the building only because he does not know how to figure the area of a triangle or is un-

able to compute the shingles required for the roof because he does not understand the simple principle of mensuration of plane figures. There is something radically wrong either with the instructor or the mental growth of the boy who, as a freshman in college, startles his instructor with the question: "Mr.——, what is the formula for the area of a rectangle? I can't remember it?"

Elementary algebra is usually omitted from eighth grade work and from high school and agricultural school entrance requirements. I believe that the elementary algebraic concepts may well be introduced in the eighth grade work in arithmetic. I have found the algebraic conception of the simple equation and the use of symbols for the unknown quantities a great help to my own children taking problem analysis in eighth grade work. I do not believe in teaching demonstrative geometry in the eighth grade nor in fact without being preceded by considerable instruction in elementary algebra but I do thoroughly believe in extensive training in those phases of arithmetical mensuration drawn from the geometrical laws for areas and volumes. At no other point in a pupil's preparation have I found so pitiful a weakness in both school and college as in the average ability to analyze the thought in a worded problem. I am aware that this matter often resolves itself into a question of syntax, of understanding of English grammar, if you please, but even so it is a fundamental need in mathematics. If the eighth grade teacher will take to heart the suggestions here offered and lay special stress on the particular topics above mentioned, he will very greatly aid the students in our agricultural high schools and schools of agriculture in mastering the necessary more advanced work in elementary mathematics. Yet some study in both elementary algebra and plane geometry is, to my mind, essential even for the elementary student in agriculture. In presenting them, however, the courses as usually given in our text books should be very carefully and fully abridged to fit the time and the need of the student so that his efforts shall not be wasted on protracted discussions and theoretical methods in lowest common multiple, highest common factor, square and cube root and other specialized topics having little useful part in the work. He should rather give his time to the mastering of the essential con-

cepts of algebra, strengthened by a rigorous course in problem analysis. Such a course will equip him not only for his geometry but for any other phase of mathematical work in elementary science that will naturally come into his course of study.

Teaching of Agriculture and Agricultural Extension

The avenue of our educational system leading to preparation for this work of teaching and agricultural extension is the College of Agriculture of the University. The preparatory school requirements for entrance to the College of Agriculture are very similiar to those for entrance to any other department of the University, mathematics not excepted; namely, a complete regular high school course of four years, including at least one year of elementary algebra through the simpler types of quadratic equations, and one year on the complete course in plane geometry as ordinarily presented in our standard texts. Students looking forward to entering College of Agriculture are further advised to take a third year of mathematics in their high school work covering half a year of advanced work in elementary algebra and half a year in solid geometry. The limitations discussed for School of Agriculture students in algebra and geometry should not in general apply to regular high school students preparing themselves for work in the College of Agriculture. Such students should in general cover the entire course in elementary algebra and in plane geometry with great thoroughness. Those students who come to the College of Agriculture with only two years of high school mathematics are required to take a special course in applied elementary mathematics intended to cover their special needs in mathematical work in undergraduate study. This course extends through a quarter of twelve weeks, five recitations per week and is the practical equivalent in effort and subject matter of a half year's work in the high school. It takes up a few of the more important topics in advanced elementary algebra, such as quadratic equations, proportion and variation, true averages and graphs, and gives in addition a brief course in the essentials of plane trigonometry insofar as they apply to the solution of triangles. The latter part of the course is a miscellaneous mass of problems of varied character relating to almost every phase of undergraduate work in the college,

special stress being laid upon farm management, dairy production, forest valuation and managment and advanced mensuration and survey problems. Those students who come from high school with a full three years in mathematics are not required to take this course or any further mathematics.

In the College of Agriculture we used to require of all boys entering the college with only two years of high school mathematics a term of Part I of higher algebra followed by a term of plane trigonometry. All freshman boys, regardless of the amount of high school requirements in mathematics, were required to take this course in plane trigonometry unless by chance they had also taken the plane trigonometry in the high school. The crowded condition of the curriculum in the college forced cutting down the time given to mathematics, resulting in the requirements above outlined. I believe the present arrangement is a mistake. We would get far better results with our students in the College of Agriculture if we could require at least three years of high school mathematics previous to entering college and still require *all* freshmen boys in the college to take this special course in elementary applied mathematics.

Scientific Investigations Relating to Agriculture

The preparation for this work, as in the case of the preceding phases, is a college course in agriculture and has to do more particularly with study along the line of special agricultural sciences, such as plant breeding and pathology, agricultural chemistry and soils, agricultural economics and rural sociology. The undergraduate work in these is largely of a preparatory character in general science and agriculture. The advanced work calls also for a considerable knowledge of physics and advanced mathematics not provided for in the curriculum of the College of Agriculture. To meet this need the mathematics and physics courses of our academic college are open as general electives for both undergraduate and graduate students wherever such may be needed.

Particular essentials in the various branches of mathematics. Let us now consider for a time in detail what should be taken up in the study of the various fields of elementary mathematics.

(a) *Arithmetic* has been pretty thoroughly discussed in the foregoing so that no further time will be given to it.

(b) *Algebra.* First fundamental principles. We must consider, of course, positive and negative numbers and the four fundamental operations, laying stress on the special products represented by the following type forms: $(a \pm b)^2$, $(a + b)(a - b)$, $(a \pm b)^3$; and the product of two binomials having a common term, $(a + b)(a + c)$.

These should be followed closely by the simpler factorable types as follows:

(1) Polynomials, each term of which contains a single monomial factor.

(2) The grouping type, such as: $am + bn + bm + an$.

(3) The perfect square of the form: $a^2 \pm 2ab + b^2$.

(4) The difference of two squares of the type form $a^2 - b^2$.

(5) The quadratic trinomial $a^2 + ax + b$.

The last three at least should be checked by visual geometric representation and demonstration.

(6) The general quadratic trinomial such as $ax^2 + bx + c$.

(7) The sum or the difference of the same powers of two quantites as $x^n \pm y^n$.

I believe the more intricate forms of factoring types should be omitted from high school courses. Then should come the algebraic concept of the fraction defined as merely an indicated, unperformed division, its manipulation therefore dependent on the more elementary laws of division. Then we should take up both simple and quadratic equations, solved by analytical methods. Lay little stress on solution of quadratics by formula which is liable to be made merely a matter of memory and so trip the student at a critical time by a lapse of memory. Carry the solution of simple equations into simultaneous equations of two or more unknown quantities. I do not, however, believe in using up the average student's time in long discussion of types of simultaneous quadratic equations. Bring in the elements of graphs just after the elements of quadratic equations and teach the student not only the graphical solution of the equation but also the depicting of statistical and other scientific data by the process of the mathematical graph, based on two rectangular axes. This is not difficult and if presented in the right manner

can be covered in a week's time in ordinary high school work. We hear a great deal of talk from time to time about proportion in algebra as obsolete. The further the advanced student goes the more he uses the algebraic principle of proportion and variation. Hence, the high school teacher of the present day would do well to forget the suggestion that the teaching of proportion as such in algebra is obsolete. If it is ommitted in the high school work, we simply have to teach it in the college. Progressions should also be taught along with variation which is essential in almost every phase of college agricultural work. I do not believe in spending a great deal of time on algebraic demonstrations of rules. That is, I think so-called higher algebra for high school students is largely a waste of time. I would give the time usually allowed for such work to an extra half year of advanced elementary algebra. Some of the best of our high schools are doing this and have been for years past with good results. If it is found impossible to cover the usual course in elementary algebra thoroughly in the one year allowed provision for this extra half year of advanced work will take care of the shortage in the freshman year and also give the student the additional topics of progression, proportion, and variation, graphs, and more extensive problem analysis, by practice rather than by abstract proofs.

(c) *Geometry.* For the agriculture school student or the high school student preparing for the business of farming, the something like 270 or 280 principal propositions ordinarily covered in our standard texts on plane and solid geometry can by careful and systematic culling be cut from one third to one half and this should be done, due to the lack of room in the crowded curriculum. If it is done the course in plane geometry can quite tolerably well be covered in one six months' year of twenty-four weeks, five recitations per week with at least one month to spare for covering the essentials of solid geometry insofar as these apply to formulas for volumes of simple geometric solids. In the ordinary high school course of nine months, five times a week, it is customary to give one year to plane geometry, and the next half to solid geometry. I think this may well be put all into one year for agricultural students. It can be done. In handling the subject of geometry, the essentials are, of course,

a clear conception of the geometrical elements, point, line and surface, as for example, define a line not merely as that which has length only but as the path of a moving point, or define a straight line, not merely as a line which does not change its direction throughout the entire course, but also as a portion of a circumference of a circle of infinite radius. That is, I would give a little more time to the elementary fundamental ideas and broaden the student's view. Then follow with perpendicular and parallel relations. These are few and definite and must be thoroughly mastered. Then comes the subject of triangles and other plane figures which can be made very brief and pointed, especial emphasis being given to the nature of the right triangle and its relation to the isosceles and equilateral triangle. We must have complete understanding of the principles of equality of plane figures, similarity of plane figures and areas of same, but let us cut out most of the mass, given in standard texts, of special original problems that lack usefulness and stick to the more practical phases. I do not mean by that that we should get away from insisting on a clear understanding and thorough memorizing of fundamental definitions, axioms and statements of theorems. That we must have. Do not insist on demonstration by the book but encourage originality and the simplest possible proofs of theorems. In solid geometry, practically all that is necessary is the elementary discussion of the plane and its relation to the line and the point, the perpendicular and parallel relations as in the case of plane figures, equality and similarity of solids, and the developments of formulas for the volumes of the simple solids. For those fundamental theorems which require more abstract reasoning but are absolutely essential to the development of volume formulas, physical demonstration apart from purely mathematical deduction is to my mind perfectly allowable and often helpful for the slow student. For example, he will probably have little difficulty in understanding the prism or the cylinder and mastering the fact that two prisms or cylinders, having equal bases, are in the same ratio as their altitudes, but when he gets to the pyramid or the cone, he may have more difficulty, for example, in proving the theorem that two pyramids having equivalent bases and equal altitudes are equivalent. I have seen the proof of this proposition abso-

lutely stump some of my best students. If the teacher can secure two such pyramids of wood or other material of entirely different shape but being sure that their bases are equivalent and their altitudes equal and immerse each of these in turn in a measuring jar of water, even the dullest student can see for himself that they displace equal amounts of water. It is also possible to build up models with strips of xylonite, wire and thread which will ilustrate all sorts of relations between lines and planes and I think this should be done where a student or group of students are slow to grasp the analytical methods.

(d) *Plane Trigonometry.* I do not advocate the teaching of plane trigonometry in the high school. The opinions of many eminent professors of mathematics in our colleges to the contrary notwithstanding, I maintain that it is not a high school subject. Nevertheless, if it must be taught in the high schools, let us stick to the bare elements and make it purely an extension of plane geometry. If we teach trigonometry, we must necessarily teach the elements of logarithms which means the use of the common logarithm table and if we are to teach logarithms, let us teach it in our elementary and advanced algebra along with the subject of exponents, showing that a logarithm is what it is, merely a special type of exponent. Do not try to explain how it is arrived at but merely how it is made up and what its action is in the ordinary operations of multiplication, division, involution, and evolution. Then in trigonometry, let the teacher pass, by as clear explanations as possible, from the similarity of plane triangles whose sides are proportional to the fundamental idea and definition of the trigonometric functions. There is no need to spend any great amount of time in the study of the inter-relations between the functions apart from the reciprocal relations which appear in the primary definitions. It becomes at once apparent to the interested student that these fundamentl definitions are the laws for the solution of right triangles and it is quite possible to give all the necessary laws for the solution of oblique triangles in five simple geometrical demonstrations covering about as many pages in an ordinary text. There is no need to drag the student through fifty or sixty pages of long analytical discussion, leading him up by slow stages to the proof of the tangent law if he has thoroughly committed the definitions of the trigo-

nometric functions and understands what they represent. Extensive solution of simple problems in triangles and other simple plane figures will fix the laws of trigonometry very firmly in the mind of the student and at the same time arouse his interest.

General Considerations. Let us be firm and concise on fundamental definitions, *e. g.*, the pupil must be clear in his disinction between a factor and a term but let us banish also difficult scientific names. If two triangles are equal under certain conditions, let them be equal, not congruent. Let us pick the shortest proof we can for difficult propostions, not the longest and most complicated ones. For example, in the comparison of plane figures and solids, having one or two similiar dimensions, let us not insist that the student must drag in, and without understanding, the abstract theorem of limits. Let us look for a moment at the theorem. "Two rectangles having equal altitudes are to each other as their bases." Assume Case I, the commensurable case, to have been proven. Case 2 is where the bases are incommensurable. Let us proceed as follows: first the divisor in the first figure, being established, it is always greater than the remainder in the second figure. Second, now continue to reduce the divisor of the first figure; the remainder in the second figure will likewise be reduced. Third, if we continue to reduce the divisor until it has become the smallest possible quantity, the remainder will then have become less than the smallest possible quantity or zero; the two figures are then commensurable and the theorem is proven by Case 1.

Conclusion: I have dealt largely in generalization. It has been my intention to show that the requirements in mathematics for agricultural study are not extensive but intensive, not complex and advanced but simple and elementary, not theoretical and abstract, but practical and concrete. It is in education today as in other things; the interesting and new theory and unexplored realm of knowledge, the intellectual luxury, if you please, of yesterday becomes the commonplace requirement of mental equipmnt requisite for success today. The result is our school and college curricula are crowded with essential requirements in many lines and the cry is constantly for the stronger and plainer, more utilitarian lines of mental architecture and the dispensing with the purely ornamental furbelows and friezes

of the past to make room for the many essentials. We mathematicians, ever conscious of the lasting worth and fundamental strength of the science, still must hear and strive to heed the demand of the hour if we would not see the mathematics temporarily lose—for it cannot lose it permanently—the place of power and strength it has so long and rightly held in the educational structure.

A COMPOSITE COURSE FOR SEVENTH AND EIGHTH GRADE MATHEMATICS*

Foreword. The territory of the Lake Shore Division of the Illinois Teachers' Association comprises some half dozen counties in the northeast corner of the state exclusive of the City of Chicago. A considerable portion of its population is suburban to the larger city and practically all of its business activities center there. Its educational experiences and problems are not peculiarly its own for all of its communities are vitally American. It does admit a large foreign element in its population but this is well distributed and eagerly striving to become American. But its compact territory, good transportation, and alert school authorities have all served to keep it well forward in striving for the solution of educational problems.

Organization of Township High Schools. With one or two exceptions, all the secondary schools within a radius of seventy-five miles of the City of Chicago are organized under the Township High School Act. This plan of organization is very popular in Illinois with nearly four hundred such districts already in existence and the number increasing everywhere throughout the state with the coming of paved roads.

Drawbacks of Township High School. But the township high school tends greatly to magnify the differences in organization and, especially, in methods of instruction that exist everywhere in this country between the elementary school and the secondary school.

(1) The elementary school and the township high school are under distinct school boards with distinct and absolutely independent teaching forces and superintendents. The high school is not responsible to the elementary school nor is the elementary school in any legal way responsible to the high school.

(2) The township, which is the high school district, usually includes several independent elementary school districts, each with its own school board, its own independent superintendent, and its own methods of meeting educational problems. If one elementary district in a given township has a reasonable popula-

* A report by the Articulation Committee for Mathematics of the Lake Shore Division of the Illinois Teachers' Association.

tion and considerable wealth, it cannot help but work out its problems in a better manner than its neighbor district in the same township with larger population and less wealth. But under the Township High School Act the secondary school must accept the diplomas from every grade school in its territory and at their face value.

Efforts to Meet Drawbacks. To bring about a better co-ordination between the grades and the high school, many township principals have adopted the plan of making some instructor who is well known and popular throughout the high school district chief liaison officer—his duty being to visit all the teachers of the higher elementary grades in their classrooms, study their difficulties at first hand, and present the problem of deficiency in preparation for the upper school. Of course this can only be done with the full approval and consent of the grade superintendent.

Some high schools, ruled out of direct contact with the grade teachers, have aroused their interest and zeal by following the college plan and sending back to each the record of the work in the upper school made by each graduate of the lower during his first semester.

One township high school of the Lake Shore Association has an articulation committee appointed from its faculty, one representative for each freshman subject, that meets all the seventh and eighth grade teachers at the high school building several times during the school year. These meetings have served to bring about a fine spirit of co-operation between the teachers of the six elementary school districts that comprise the township and the high school. Minimum requirements have been agreed upon for the important grade subjects, notably English and mathematics. Some of the common errors made in the upper school have been listed and some study made of their causes. This particular school has this year, for the first time, attempted to classify all first year students on the basis of the Otis Tests given last spring in the grades together with the records of the grade teachers and their estimates of ability. It has placed all slower pupils in distinct sections so as to enable them to proceed at their own speed. The reports from such grading should prove interesting.

Such a committee should get on with the problems of students' errors and what should be the carry-over from grades to high school.

Elementary Schools Awake. The elementary school authorities are fully abreast of the problem of better preparation. Each year an increasingly larger percent of their graduates enter the upper school. Of forty-five who received diplomas from one grade school in June, 1921, forty-three enrolled at the local high school in September. A similar record will be found in many other communities. Professional pride requires that the standard of preparation shall be as high as possible.

Growth of Departmentalized Grade Schools. In northeastern Illinois there has been a steady growth in the number of departmentalized seventh and eighth grade schools. The Township High School Act which restricts the district thus organized to the control of ninth, tenth, eleventh and twelfth grades only is apparently compelling the adoption of the 6-2-4 plan. Such two-year schools are to all intents junior high schools of two grades. Several new buildings and remodeled old buildings have been entirely given over to the use of such schools. In a large number of elementary districts the seventh and eighth grade pupils have been assembled from several contiguous wards in suitable rooms of a single convenient building. In smaller elementary districts that have but a single building the problem of departmentalizing these grades has been easier. In every district able to pay the bill two or three years will see the reorganization completely worked out. But of course the problem of teacher-preparation and of subject material will require a longer time. Whatever may be the actual advance in quality of preparation for high school due to this movement, at the present it is stirring the enthusiasm and arousing the professional spirit of the teachers. They are striving to become specialists and are getting away to the summer schools of the universities in increasing numbers.

An Articulation Committee. At the annual meeting of the Lake Shore Association in April, 1921, the newly-elected president, Principal E. V. Tubbs, of New Trier Township High School, named an Articulation Committee for mathematics consisting of ten grade and four high school teachers of the subject.

The membership of this committee was selected with reference to a central meeting place and represents seven grade districts and four township high schools. To it was assigned the problem of better co-ordination. It is interesting to note that President Tubbs has recently named a similar committee for English.

At its first meeting in June the mathematics committee decided that its first duty was that of making a composite course that could be adopted with minimum changes throughout the Association. In the immediate suburbs of Chicago there is a constant movement of population and a considerable interchange of pupils during the year. The problem of adjustment to a new school is sufficiently difficult for the ordinary student even when the subject material is quite the same in the new and the old. But when the courses differ as widely as at present in some sections, the adjustment is impossible and a year is lost.

The Composite Course. The following is the composite course agreed upon by the committee. Few schools will need to make any very great change in their present material, time, or manner of its presentation. Furthermore, while not extreme or radical in any way, the committee believes that it holds close to the similar report of the National Committee on Mathematical Requirements.

MATHEMATICAL TOPICS FOR THE SEVENTH GRADE

At the completion of the sixth grade the pupil should be able to perform the fundamental operations with integers, common fractions, and decimals with a fair degree of speed and accuracy.

I. Review briefly essential topics of previous grades for accuracy and against future needs.

II. Percentage and its applications.
  1. Its three cases.
  2. Change per cents to fractions and vice versa.
  3. Choosing problems
     a. arising from the home and community.
     b. introducing industry and business.
     c. teaching thrift and economy.

III. Intuitive geometry and simple mensuration.
  (All geometric terms to be carefully defined and correctly used.)
  1. Construction work with compasses and straight edge.
     a. Drawing straight lines and circles.
     b. Bisecting line segments and arcs.
     c. Constructing and measuring angles.
     d. Constructing perpendiculars.
     e. Constructing parallel lines.
     f. Reproducing and originating designs and patterns.
  2. Constructing and using squares, rectangles, parallelograms, triangles, etc., measure lengths, compute areas, and derive the simpler formulas. (The formula as a shorthand.)

IV. Simple interest and its problems.
   1. Introduce and use formula, $i = prt$.
   2. Use business terms and methods.
   3. Solve investment problems from real estate, postal savings, etc.

V. Graphs—their interpretation and construction.
   1. Bar graphs.
   2. Circle graphs.
   3. Motivate 1 and 2 with simple problems requiring the expression in percents or fractions of relative areas of given graphs.

MATHEMATICAL TOPICS FOR THE EIGHTH GRADE

I. Review fundamental operations with integers, common fractions and decimals for speed and accuracy.

II. Complete the study of percentage and interest.
   1. Successive discounts and other business terms.
   2. Banking and its papers.
      a. Savings bank interest.
      b. Bank discount.
   3. Use problems
      a. Of investing money, loans, mortgages, stocks and bonds, etc.
      b. Teaching the terms and meaning of insurance.
      c. Teaching the meaning and necessity of taxes.
      d. From internal revenue, tariffs, etc.

III. The Pythagorean Theorem and square root.
   1. Explain the theorem by squares from cross section paper.
   2. Teach method of square root and require reasonable accuracy.
   3. Apply the theorem as widely as possible, diagonals, altitudes, etc.

Note: Topics IV, V and VI are to be taught in a unified manner.

IV. Mensuration.
   1. Review work of seventh grade and construct regular polygons of 6, 8 and 12 sides.
   2. Review and complete essential tables of denominate numbers.
   3. Board measure and its formula.
   4. Surface and volume.
      Derive formulas for and apply to problems arising from (a) trapezoid; (b) circle; (c) equilateral triangle; (d) prism; (e) cylinder; (f) cone; (g) pyramid; (h) sphere.

V. Simple equations.
   1. Finding value of unknown.
   2. Four operations performed on an equation.
   3. Solving problems by equations.

VI. Formulas.
   1. Evaluation of.
   2. Changing subject.
   3. Application to problems.

Note: Topics VII, VIII and IX are to be taught in a unified manner.

VII. Ratio and proportion.
   1. Meaning of ratio.
   2. Meaning of proportion.
   3. Application to problems.

VIII. Similar figures.
1. Discovery of relationships.
2. Development of idea by problems.
3. Use in drawing maps and plans.
4. Measurement of inaccessible distances.

IX. Graphing.
1. Review of seventh grade work.
2. Problems requiring representation of ratios and percents by graphs.
3. Construction and interpretation of broken line graphs.
4. Showing functional relation by graphs.

At its furture meetings the committee proposes to fill in the outline by naming minor details necessary for perfect development. For instance, it proposes to name what formulas shall be presented and in what order, what types of problems can be used from life and industry and when, what graphs had best be required and along what lines can exploration be made most easily by the pupil.

It proposes to get at the problem of consistent use of terms, for it finds the complaint that the pupil is unable to distinguish between "gain" and "rate of gain," between "sum," "amount" and "principal," etc. It finds that most texts are not consistent in the use of certain expressions. One problem may read: "A sum of money will amount to................" and a little farther on the pupil will find this: "Two men form a company and their capital amounts to......................." While the number of such inconsistencies in arithmetic cannot be large, yet why should there be a single case?

Articulation Committee for Mathematics,
Lake Shore Division, Illinois Teachers' Assn.

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THE TEACHING OF MATHEMATICS: THE NEED AND THE METHOD

By Professor FRANK B. WILLIAMS
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The *sine qua non* in mathematics in *accuray*. Sylvester said "The fine arts are four: Plastic, Music, Lyric and Mathematic," and Dr. Story has recently defined mathematics as "The Art of Exact Thinking.

Never, in the history of the race, was the art of exact thinking more needed than it is today; the world is full of cranks with half-baked theories of government, religiion and morals,, as well as education itself, based on inaccurate deductions from inadequate and false data, influenced by the prejudice of ignorance and the habit of illogical thinking. Moreover, they send forth their products with fanatical vehemence and their command of language is altogether too great.

It is doubtless high time for *real educators* to think and think with accuracy; for what shall it profit a man (and his race) if he gain the whole art of expression and lose the art of forming true ideas; yea, rather, the world is the loser when, by artful expression, false notions can be sent forth clothed in a pleasing and engaging garment of language, as with authority. It were far better that a man be robbed of his power of expression, if that which he wishes to express be false, because of his inability to think with accuracy. I never think of accuracy of expression without recalling a story told by Mr. McElroy in his lecture on "Some Interesting Inaccuracies." "In one of up-state cities in New York State, a prominent citizen died and the editor of the local paper (writing in his best style) wrote, 'The King of Terrors never entered a happier home;' the compositor, looking at the letters inaccurately formed by the editor, set up the type to read 'The King of Tunis, never entered a happier home;' but as such, it could never get by the all-important proofreader, who, being a man of much learning said at once, 'This will never do; the ruler in Tunis is not called a King—he is the Bey of Tunis;' so the obituary went forth, 'The Bey of Tunis never entered a happier home.' " The "cock-sure advice on how and

when to teach mathematics, given by those who never teach it, is probably compounded by some such formula as that which produced "The Bey of Tunis."

The great development in science, in all directions, was made (and is being made) possible by the development of mathematics, which in turn owes its advance from time to time, to the introduction of new and better symbols, so that mathematics is often defined as "symbolic thought;" mathematics is indeed a language, if one defines a language as a means for the expression of ideas; moreover, mathematics is a universal language, being practically the same the world over, and is therefore of more importance than any one language. Mathematics contributes not only to the great material prosperity and commercial activity, but also to the general moral and spiritual uplift by the introduction of breadth of view, depth of thought and sense of truth.

I agree with Myers in his "seventh principle," as stated in the February, 1920, number of the *Mathematics Teacher,* that good high school mathematics and good mathematical teaching are chiefly beneficial to the public school pupil anywhere, because, and in so far as, they beget and foster the habit of taking a rational attitude toward problematic situations; i. e. the habit of basing conclusions on the underlying facts." This, he says, "Is the chief reason why we cannot afford to allow the mathematical element in education to be reduced to the minimum essentials."

Mental discipline is of first importance in education and today we seem to be in danger of losing most of it through a general attitude of relaxing all discipline. The denials of the disciplinary value of mathematical training and the claim that it does not furnish any mental power that can be transferred, have been based on insufficient data, giving negative results, while the fact of increased ability due to mathematical training has impressed itself on keen observers for many centuries: certainly as long ago as the time of Plato and other Greek philosophers, as pointed out by Cajori in his article printed in the "*Mathematics Teacher*" for December, 1920, entitled "Greek Philosphers on the Disciplinary Value of Mathematics," in the course of which he quotes the following statement from Plato's

"*Republic*": "In *all* departments of Knowledge, *as experience proves,* any one who has studied geometry is infinitely quicker of apprehension than one who has not." Note the word, *proves;* when an ancient Greek used the word proves no one can doubt that the facts carried conviction to his mind in no uncertain terms. (Revisionists will please note, also, the word *geometry* used in this connection). Cajori ends his article with the statement "Philosophers of great eminence, like Plato and his disciples placed extraordinary emphasis upon the mind-training value of mathematics."

It must also be borne in mind that it was the study of geometry that first awakened to activity such minds as those of Newton and Einstein.

To me, one of the great contributions of mathematics to mind training (when properly taught) is the formation of good habits of thought and of expression which are the very first essentials for a student in mathematics; habits are presistent and certainly every child must be trained to possess the habit, as Myers says, "of taking a rational attitude toward problematic situations''' with which every citizen will be continually confronted.

There can be no doubt of the need. The great difficulty has been and is likely to be with the methods used in *teaching* mathematics, *especially* in the grades, where the first habits, good or bad, are formed. I feel quite sure that *good teaching* (the best possible) is of more importance in the grades than at any other level ,and is far more difficult work than is necessary in later years, especially if the work is well done in the grades so that correct habits are formed. We ought to insist on thoroughly trained teachers of *unusual* ability for the grade schools and pay them what they are worth—probably more than is paid to teachers in high schools, or at least as much— since their training would require as much time and effort as that expended by the high school teacher.

If the present methods of some so-called educators prevail the "viscious circle" will soon bring us to the lowest depth, if indeed we have not almost reached that level now in the grades:— First of all many pupils now get through the grades with very little mathematics—a very little working knowledge and no comprehension of the fundamental processes; second, the so-called

educator says "Well its too bad to burden these promising citizens with such stuff as algebra and geometry; these are especially annoying and burdensome to girls who have no natural ability in this direction anyway, so it will be better not to require them to take mathematics, or at least not very much;" third, today practically all of our teachers for the grades come from these same high school girls, after they have had a "Normal School training" where, however, they are taught methods of teaching but not the things to be taught; so they begin their teaching of mathematics with less knowledge than their former teachers and the downward march continues.

In the days of "long ago" when Normal Schools were first invented (at least in one with which I was acquainted in the 80's) the substance of the subjects was thoroughly given by well trained teachers and it was thought essential to know the subject before trying to teach it to some one else.

In many other professions and trades, essential to the life of the community it is necessary that those who enter them shall have the habit of performing the elementary operations of mathematics with *accuracy;* e. g. in the past few months I have been informed by one who for many years had charge of the training of nurses at a well known hospital, that the girls who came for the training could not, in most cases, handle simple fractions, such as $\frac{1}{2}$ of $\frac{1}{4}$, so essential in their work, or to get a dose of 1-120 of a grain when only 1-60 of a grain tablets were available—they were just as likely to say "give two tablets" as to say "divide the tablet into two parts."

I wonder how many of us are willing to run the risk of being given four times the dose ordered by the doctor, in a serious case.

As teachers of mathematics and as good citizens, we ought to *insist* in some way—yea, in every way—that *all* teachers who are to be intrusted with the teaching of mathematics in the public schools *shall have, at sometime, a thorough course in the subject matter under a skilled teacher with a vigorous insistence on accuracy and the proper use of symbols. Why* should anyone be permitted to teach mathematics who has no more regard for symbols than to teach her pupils (or, at least, allow them) to write $100 - 10 = 90 - 10 = 80 - 10 =$ etc.," until finally $100 = 0$? Yet this is not only done in our schools, but judging

by the work of the freshman in college, this and similar flagrant misuses of symbols are tolerated or may be even taught, not only in the grades but in the high schools.

The proper use of symbols is the very *soul* of mathematics and must be insisted upon from the first grade. The lack of a proper set of symbols to represent numbers, kept the Greeks from making an advance in methods of computation and algebraic analysis, at all comparable to their great work in geometry.

Mathematics has nothing to do with the subject matter involved; its concern is with the operations performed and may be applied to any subject provided its symbols can be given a meaning in that subject. A child must be taught that as soon as a problem has been stated in mathematical form, whether in symbols or arithmetic or algebra, or what not, then he may proceed with the allowable mathematical operations regardless of the things for which these symbols stand; only in the final interpretation of results must he pay attention to the things represented by the symbols. (It is not necessary, for example, for him to think of a ferry-boat transporting a number of cows from one side of the river to the other, when he wishes to transpose a term in x which at the outset was used to represent so many cows). To be sure, close connection with concrete things must be maintained at the beginning, for the child must use "pegs" in his mind on which to hang new ideas. The utility of mathematics must ever be kept before the pupil so that he may always be eager to learn the symbols and operations in order to profit by their use, and it must be made plain to him that his operations with symbols will be made easier and more effective if he is able for the time to forget (or disregard) the things for which the symbols stand.

Instead of a move to shorten the required course in mathematics there should be a tremendous move to lengthen the course, changing its form and content. Unlike the "Dead Languages" with which it was formerly closely and honorably associated, mathematics is forever very much alive; it serves not only in the capacity of scout and guide in pointing the telescope and microscope but it must be relied upon as the heavy artillery in the final attack. As a living, growing art, the ways and means of its presentation must be subject to change. In the last twenty

years, many changes have been made in the mathematical curricula of England, France and Germany. France has *extended* the minimum of mathematics required of all students in the Lycees, to the "elements of calculus" (where it should be). What have the changes done for us, in America? How shall we meet the vital assault now being made to minimize the usefullness of this great instrument for service in the field of education and in every walk of life and how shall we utilize to the best advantage the time now allotted to mathematics in the curriculum?

We, of the colleges, complain that our pupils come to us scarcely half prepared and the teachers of high schools say that they can do no better because their pupils come from the grades with no ability to use the four fundamental processes or to solve simple problems. Let the grade teachers speak for themselves. A few years ago I sent out a "questionaire," and from about 150 replies, from all parts of the United States, I extract the following information, as a kind of summary, from the answers to the two questions on "Difficulties encountered" and "Reforms suggested."

1st Grade

Field too large—too much abstract work and not enough work with objects.

2nd Grade

Work too hard for the child at this age; too much work with abstract numbers. Reforms suggested are: less work for this grade so that better work can be done—leave out written problems and give more drill on "combination and separation" of numbers (addition and subtraction of small numbers).

3rd Grade

Difficulties:—Inaccuracy—lack of power to think:—rush for answer without thinking of a method of solution;—lack of knowledge of combinations of numbers to 20;—lack of drill in lower grades. *Reforms*—more time for drill in fundamental processes and less variety of subjects;—more mental work and more thorough work in lower grade.

4th Grade

Difficulties:—Poor in numbers;—lack knowledge of multiplication table. *Reforms*:—more drill on tables and more practical problems;—teach pupil to test his work.

5TH GRADE

Difficulties;—Inaccuracy the main stumbling block throughout;—deficient in fundamental processes.

Reforms;—Lessen the field of work to get *accuracy* and drill; —confine work in lower grades to the four fundamental processes. Problem should be pictured by pupil before solution is attempted.

7TH GRADE

Difficulties;—*Inaccuracy* the chief fault; deficient in reading and writing and the four fundamentals;—*carelessness;*—children immature for this work.

Reforms;—More mental work;—do not teach a smattering of everything in each grade;—obtain better grounding in fundamentals;—have problems with simple relations;—insist on thoroughness and accuracy.

8TH GRADE

Difficulties;—*Inaccuracy;*—lack ability in mental calculations and ability to interpret problems;—too many subjects covered in lower grades.

Reforms;—Abolish spiral system;—omit impractical problems with large numbers;—teach more clearly the relation of numbers.

While some of the answers were a little queer (one teacher saying she "loves to teach abstract work because its tangible" and another "they have been well taught but are very inaccurate") nearly all answers were to the point and the one distinct cry, uttered by 90% of the whole lot above the 2nd grade, is "*inaccuracy.*" ("So say we all of us"). It seems quite conclusive that too much is attempted in the lower grades and that accuracy there is sacrificed for speed, which is a fatal mistake. "Safety-first" in mathematics means "accuracy first" and speed aftewards. A great deal of complaint about "Problems too hard" and "too much work in this grade," would be removed in the upper grades if accuracy was insited upon at every step and no advance granted without it (at least 90% accuracy in fundamental operations, and computations).

So, here we are, at different levels, each looking to the one below and shouting for better work and more of it, and often so

loudly that we cannot hear the cry to us from the level above. Will we make any substantial progress and meet successfully the present assault unless we study the whole situation from the bottom upwards? Can the present "National Committee" finish its work on Secondary Schools without a look at its foundation? Let us "slide down the bannister" together or take a quick elevator for the bottom and then make our way cautiously and laboriously back, lending a hand to the workers at each level, if possible, by learning their difficulties and all pulling together to remove the obstructions to progress, if any, imposed by Supervisors, Superintendents, and Boards or by an ignorant public opinion.

NEW BOOKS

Modern Applied Arithmetic. By R. R. NEELY and JAMES KILLIUS. P. Blakiston's Son & Co. Philadelphia. Pp. 156.

This book was developed primarily for part-time or continuation schools. It "is based on the unit project method," consisting of 80 "unit projects," one of which is reprinted in full here.

A BUS-LINE

In many places where the street car service is poor, or its price too high or where there is no good train service, the autobus or "jitney" business has been very profitable. This business requires quite a large capital, and is rather dangerous, so that a number of men usually form a company when they enter it. Before going into the business, a careful study is made of the possible number of people to be carried and the estimated expenses are made large enough to take care of accidents. If the outlook is good, it is usually easy to secure the money necessary for starting.

Among the expenses and receipts of such a business, the following items are most important:

Expenses.

Equipment—2 Auto Busses....	$2000.00 each.
1 set spare tires......................	60.00 per tire.
Tools and small equipment...........	150.00
4 Drivers	70 cents per hour.

Each driver works 8 hours per day except on Saturday and Sunday, when he works 9 hours.

Oil, each bus uses 1 gallon of oil per day at 60 cents per gallon.

Gasoline, each car uses 6 gallons gasoline per round trip at 28 cents per gallon.

Repairs and accessories.............	$ 90.00 per month.
Rent for garage.......................	$ 20.00 per month.
Licenses	$100.00 per year.
Advertising	$150.00 per year.

One new set of tires for each car every three months at $240.00 per set.

Conditions.

Each car makes 16 trips or 8 round trips daily except Saturday and Sunday; then 18 trips or 9 round trips.

Average number of passengers per car per trip daily = 7.

Average number of passengers Saturday and Sunday = 10.

The cars run 52 Sundays and 52 Saturdays each year.

Charge for passengers 40 cents per trip.

QUESTIONS

1. How many single trips does each car make per year? Both cars? Round trips?
2. What does the gasoline cost per year for both cars?
3. What does the oil cost for both cars per year?
4. What do the tires cost per year for both cars?
5. How many passengers are carried by each car daily? Saturdays and Sundays? How many passengers in all? How much money is received from the passengers per year?
6. How much money is paid to the drivers per year?
7. Find the total expenses for the year. Add 10% for unforeseen or extra expense.
8. Find the profits.

The authors have set up concrete, human situations which induce the *problem-attitude* in the pupil. The pupil feels that the book is addressed to him. It is a truism to say that such material elicits greater interest or that it develops a greater sense of value in the learner. What a gain if regular texts in mathematics could be constructed in this manner!

THE CHICAGO MEETING OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Teachers of mathematics throughout the country should be interested in the Chicago meeting of the National Council of Teachers of Mathematics not only because of the program which has been arranged but also because the success of this meeting will have a vital influence on the future development of the organization. Teachers should go to the meeting ready to discuss plans for the future of the Council.

Does the constitution printed below need amendments?

What constructive work should be undertaken by the Council during the next year?

What can be done to interest local organizations in the work of the Council without impairing the work of the local organization?

If you have suggestions along these or other lines and cannot attend the Chicago meeting send them by mail to the President, J. H. Minnick, University of Pennsylvania, Philadelphia, Pa.

The dinner in the Palmer House on the evening of March 1st should be a big success. Those planning to attend should send their reservations to Mr. C. M. Austin, Oak Park High School, Oak Park, Ill. You are urged to make reservations as early as possible.

Special railroad rates may be had by all members of the National Education Association if they first apply to J. W. Crabtree, Washington, D. C., for identification certificates.

CONSTITUTION OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

I. NAME

This organization shall be known as the National Council of Teachers of Mathematics.

II. OBJECTS

The purposes of this organization shall be:

1. To secure a greater degree of co-operation and solidarity among the teachers of mathematics.

(*a*) To provide for the wide publicity of important reports and addresses related to mathematics and the teaching of mathematics through an official organ and other publications.

(*b*) To vitalize and co-ordinate the work of the many organizations of mathematics teachers throughout the country.

2. To bring the interests of mathematics to the attention and consideration of the educational world.

III. TIME AND PLACE OF MEETING

Regular meetings of the Council shall be held annually in connection with the meeting of the Department of Superintendence of the National Education Association.

IV. MEMBERSHIP

Membership in the Council shall be of two kinds: individual and collective.

All persons who are in sympathy with the work of the Council shall be eligible to individual membership.

All organizations of mathematics teachers shall be eligible to collective membership.

The annual dues of the individual members shall be two dollars.

The minimum dues for collective membership shall be three dollars for each organization of less than fifty. For each additional hundred members, or fraction thereof, the dues shall be an additional five dollars.

V. MANAGEMENT OF THE COUNCIL

1. *Officers.* The officers of the Council shall be a President, a Vice President, and a Secretary-Treasurer. Their duties shall be those commonly pertaining to these offices. The President and Vice President shall be elected for a term of one year. The Secretary-Treasurer shall be elected for a term of three years.

2. *Executive Committee.* There shall be an Executive Committee of nine members, three of whom shall be officers. The remaining six shall be elected, two each year, to hold office for a term of three years; except that at the first meeting, two shall be elected for one year, two for two years, and two for three years.

The Executive Committee shall manage the business of the Council, authorize the appointment of committees and fill vacancies in office.

3. *Nominations.* At each annual meeting the President shall appoint a Nominating Committee of three members who shall report nominations for officers and for members of the Executive Committee.

VI. AMENDMENTS

This Constitution may be amended by a two-thirds vote of the members present at any regular meeting.

Program of the Chicago Meeting of the National Council

President J. H. Minnick announces the following program for the Third Annual Meeting of the National Council of Teachers of Mathematics, to be held in Chicago, March 1, 1922, during the week of the N. E. A.:

1. Business meeting, 10 A. M.
2. Afternoon program, 2:30 P. M.
   - The Function Concept in High School Mathematics, Dr. Jacob M. Kinney, Hyde Park High School, Chicago.
   - Elective Courses in Senior High School Mathematics, Professor Earl R. Hedrick, University of Missouri.
   - Some Phases of the Work of the National Committee, Professor J. W. Young, Chairman.
   - Some Problems in Secondary School Mathematics, Mr. Alfred Davis, Soldan High School, St. Louis, Mo.
3. Evening program. Dinner, 6 P. M., Professor H. E. Slaught presiding.
   - The Case for General Mathematics, Principal W. D. Reeve, University of Minnesota High School.
   - Reaction versus Radicalism in Secondary Mathematics Teaching, Professor George W. Myers, University of Chicago.
   - Is the Teaching of Mathematics Responding to Modern Demands in Secondary Education? Raleigh Schorling, The Lincoln School of Teachers College.
   - A Program for the National Council of Teachers of Mathematics, President J. H. Minnick, Dean of the School of Education, University of Pennsylvania.